5 waves

Waves allow us to receive television and radio signals and watch live telecasts of our favourite events half way across the globe, see the light from distant stars, see the minute structures in a cell in our body, send text messages to our friends, detect oil and gas hidden far below the surface of the earth, and communicate all this to each other!

Simply put, a wave is a rhythmic undulation that moves in space with a finite speed. A periodic movement of one part causes nearby parts to start oscillating which in turn incite its neighbors, and so it continues, thereby creating a wave.

Contrast this with pure oscillation, which is just periodic motion of any object or any quantity in time, without the propagation in space at finite speed which are the main defining characteristics of a wave. (Think of the wave of motion of cars as the light turns green at a road signal – the first set of cars start moving, then after a delay the next set behind them moves, and so on. If only most drivers realize that the speed of such a wave is much less than the speed of the green light waves which instantly reach the drivers' eyes, much of the honking at the signals will reduce drastically – a real example of how understanding of mathematics may change our everyday lives!)

Where do waves occur? Everywhere – literally! Wave phenomena is ubiquitous: waves on a still pond, in the deep ocean producing tsunami, in the atmosphere, in the dense plasma in our Sun, the light that

lets us see, the sound that we hear, the patterns on the tabla. The list is only limited by our imagination – literally!

What's the mathematical description of these phenomena? All waves have a wavelength – the distance after which it repeats itself; A frequency – the number of times it repeats itself in a second; An amplitude – how loud it is; And a phase – where does the wave start from? Can you notice the similarities and the differences with the characteristics of an oscillation? The equation for the undulating pattern of the wave is written in terms of some quantity W that is varying periodically as a function of position x and time t. For example, W could be electric field (e.g. for light), variation of pressure (e.g. for sound or plasma waves), or distance (e.g. waves on a spring). The simplest of such oscillations is given by

(3) $$W(x,t) = A\sin\left[2\pi\left(\frac{x}{L} - ft + p\right)\right].$$

Here, L is the *wavelength*, f the *frequency*, p is the *phase*, while A is the *amplitude* of the wave. Another important quantity is the *wave speed* $c = Lf$. Exactly as in the case of oscillations, any wave, however complicated, is simply a sum of waves of various frequencies, amplitudes, and phases.

Notice this looks deceptively like the equation for an oscillation which depends only on time t, but whereas the waves depend on the position x and time t.

Just like in the case of the oscillation, the above equation for waves is itself a solution of many different types of equations, all of which are partial differential equations. The most basic of these equations describing the waves is, not surprisingly, the wave equation!

(4) $$\frac{\partial^2 W}{\partial t^2} = (Lf)^2 \frac{\partial^2 W}{\partial x^2}.$$

A large number of other equations – those describing light, or electrons, or flow of air and water, or traffic, and many others – can be transformed to describe waves. This is the power of mathematical modeling of the world around us: it affords a unified description of a diverse set of observations!

There are several exhibits which show different manifestations of these phenomena of waves. Let us discuss them one by one.