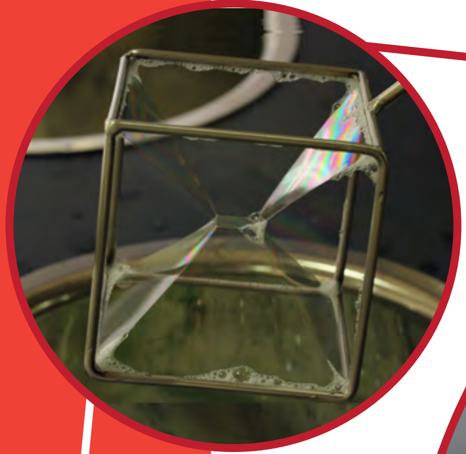
Soap Bubbles

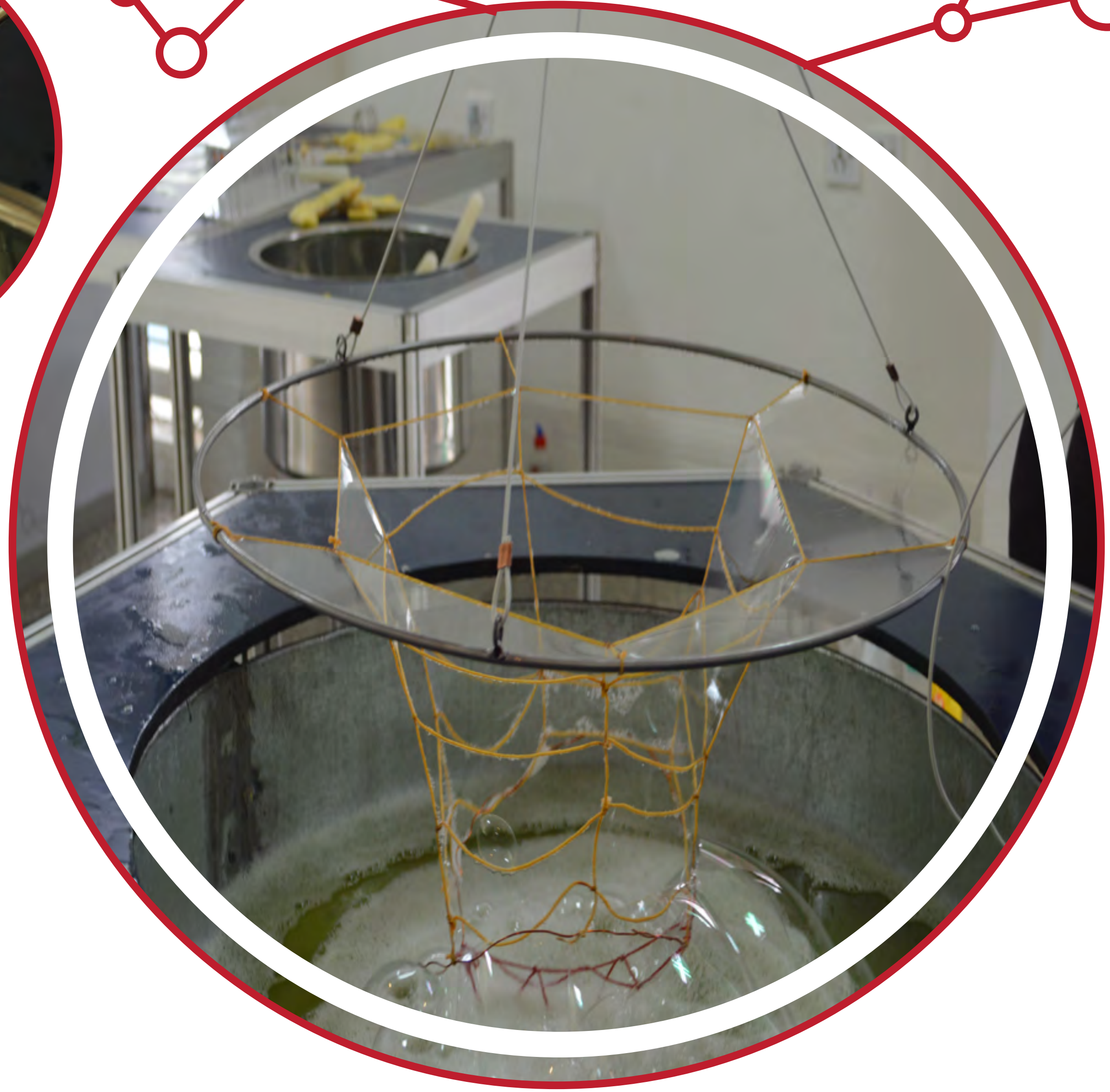
Soap bubbles are formed by the simple physical principle of surface tension, whose mathematical manifestation is the concept of "minimal surfaces" – the idea that soap films always minimize their surface area.

This seemingly simple principle leads to visually appealing and mathematically interesting shapes: individual bubbles are spherical in shape maximizing the volume enclosed, the bubble clusters settled on a plane surface display a hexagonal tessellation, the soap films formed by irregular shapes can show a dazzling variety.

Mathematically, some of the soap bubbles are classic examples of surfaces with negative curvature, like a saddle, while others are classic examples of surfaces with constant positive curvature, like the spherical bubbles. Soap bubbles have led to some intriguing mathematical discoveries as well, as we will see below.

In a state of equilibrium, the surface tension on a soap film is the same at all points, and its surface area will have a minimal value; this minimum area property of soap films can be used to solve some mathematical minimization problems.





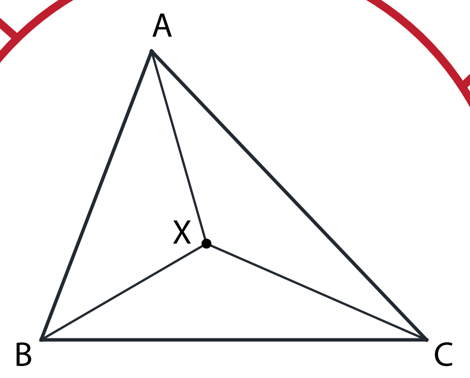


Figure 1. X is the centroid of the triangle ABC.

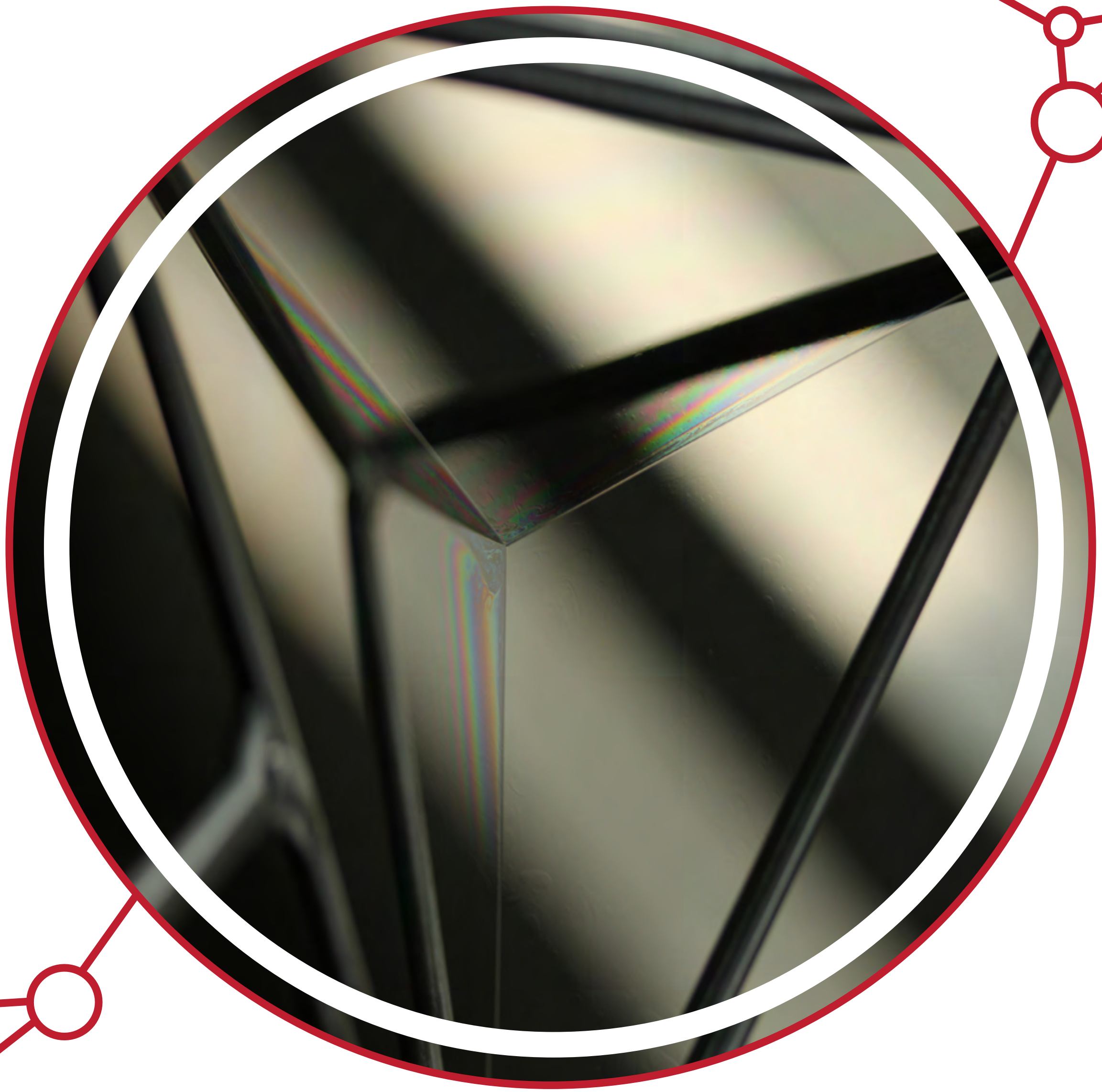
One such minimization problem is the problem of linking n points on a plane by the shortest possible path. For example, consider linking three towns A, B and C situated at the corners of a triangle by a road. What would you think is the shortest path linking the three towns? Or, what about four towns situated at the four corners of a square?

Such shortest paths are along straight lines forming a number of intersections: each intersection always contains three straight lines meeting at angles of 120 degrees. Figure 1 on the left shows a simple minded solution for linking three points at positions A, B and C that the intersecting soap films achieve. Suppose x is the location of the intersection. Then we basically want to minimize the sum of length of these three line segments, which is written as follows.

Find x that minimizes $f(x) = \|x - A\| + \|x - B\| + \|x - C\|$

The critical point (that is, the minimum) is given by $\nabla f(x) = 0$;

$$\frac{x-A}{\|x-A\|} + \frac{x-B}{\|x-B\|} + \frac{x-C}{\|x-C\|} = 0.$$



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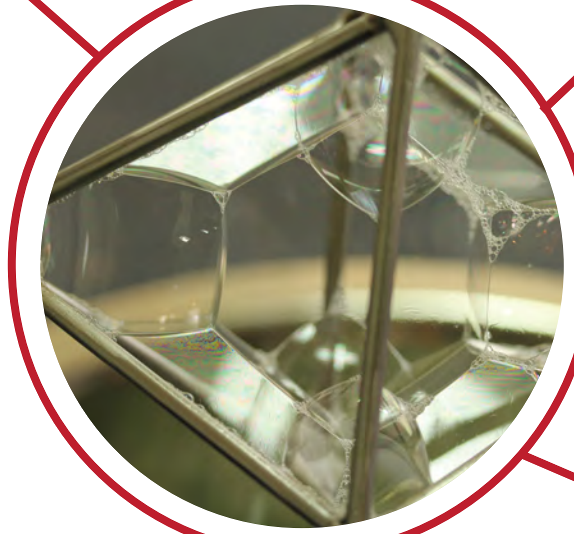

This means that we have three vectors of unit length adding up to zero. Connecting the vectors head to tail means that they form a cycle, which is a triangle. Since they all have the same length, the triangle is equilateral. Thus we see that the angle between the lines is 120 degrees.

Some of the soap bubble frames in this exhibit demonstrate this phenomenon. Even the surfaces of soap films always seem to meet at 120 degree angle, but this mathematical problem is still a challenge to researchers.