6.2

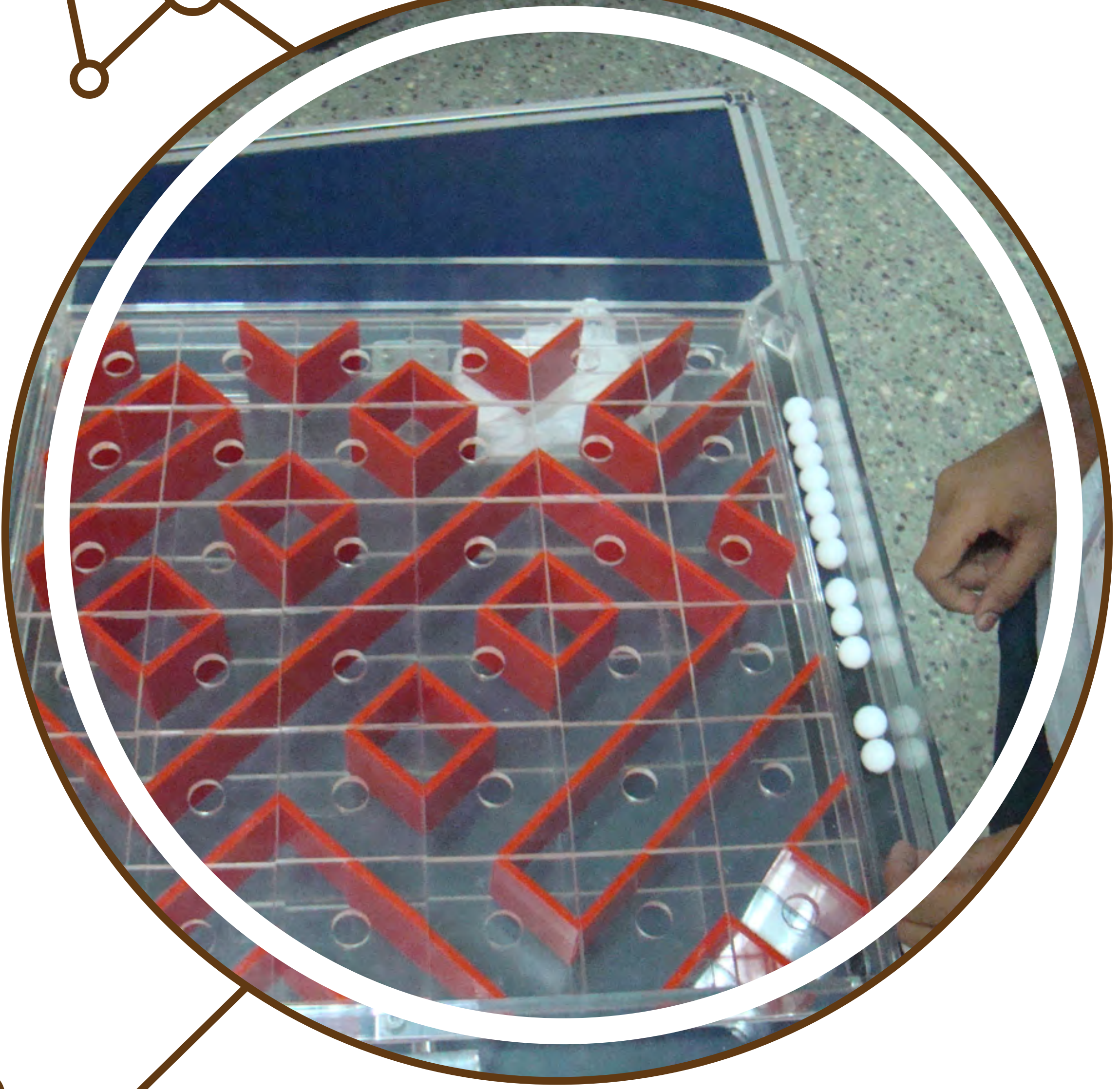
Permeability/ Percolation

As noted above, the sample space (which is the collection of all possible outcomes of an experiment) corresponding to the experiment of tossing a single coin is simply $\{H, T\}$, where H = Head and T = Tail. However, if we toss two coins then the sample space becomes $\{HH, HT, TH, TT\}$, whereas the sample space for 3 coin tosses is $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$. What if we toss 36 coins?

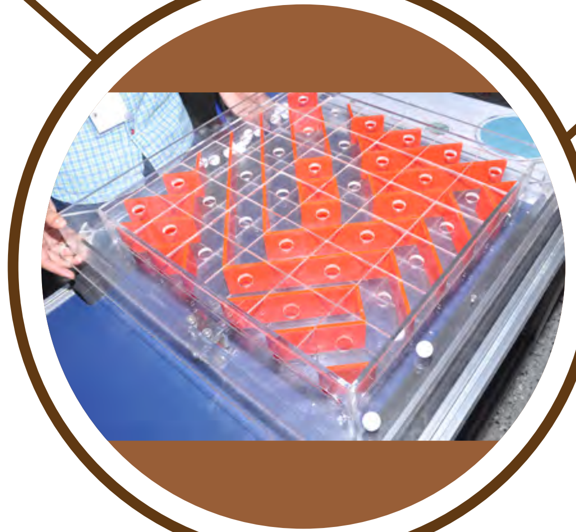
This was precisely at the heart of an exhibit which has 36 small squares arranged in a 6 × 6 pattern inside a large square. Each smaller square had a path going through it along its diagonal (any one diagonal). Each of these smaller squares could be turned by 90°. So effectively, each square has two distinct orientations. Now what are the total number of such configurations for all the 36 squares together?

The relation to coin toss is as follows: suppose we say that a head of a coin toss will correspond to the path going towards top left while a tail of the coin toss to the path going towards top right. So the configuration of all the 36 squares can be determined by throwing 36 coins and arranging the squares according to the outcome of these tosses.

What is the math? Try answering these questions for a smaller experiment, say with 3 × 3 square, or 2×2 square for that matter. Notice that the answers are hard to come by as the number of squares grow from 2 to 3, or from 3 to 4. Such combinatorial questions are at the center of contemporary research in *probability theory*.



6.2



Where do such problems occur? Have you thought about how the water seeps through loose soil or a sponge but not through a hard stone? Think of the balls rolling down through the squares in the exhibit as water going through a rock. Whether the rock is permeable or not, that is whether the water can seep through the rock or not, is determined by whether it can find a path going from one end to the other. This is the phenomenon of percolation leading to the property of permeability. The simple probabilistic experiment in this exhibit illustrates this idea. Can you arrange the squares in a way so as to make this "rock" permeable or impermeable.

Finally, going back to the idea of the coin toss, how many of the combinations will lead to a permeable exhibit while how many will lead to an impermeable one? If you were using coins to decide the arrangements of these squares, will you be more likely to come up with a permeable arrangement?