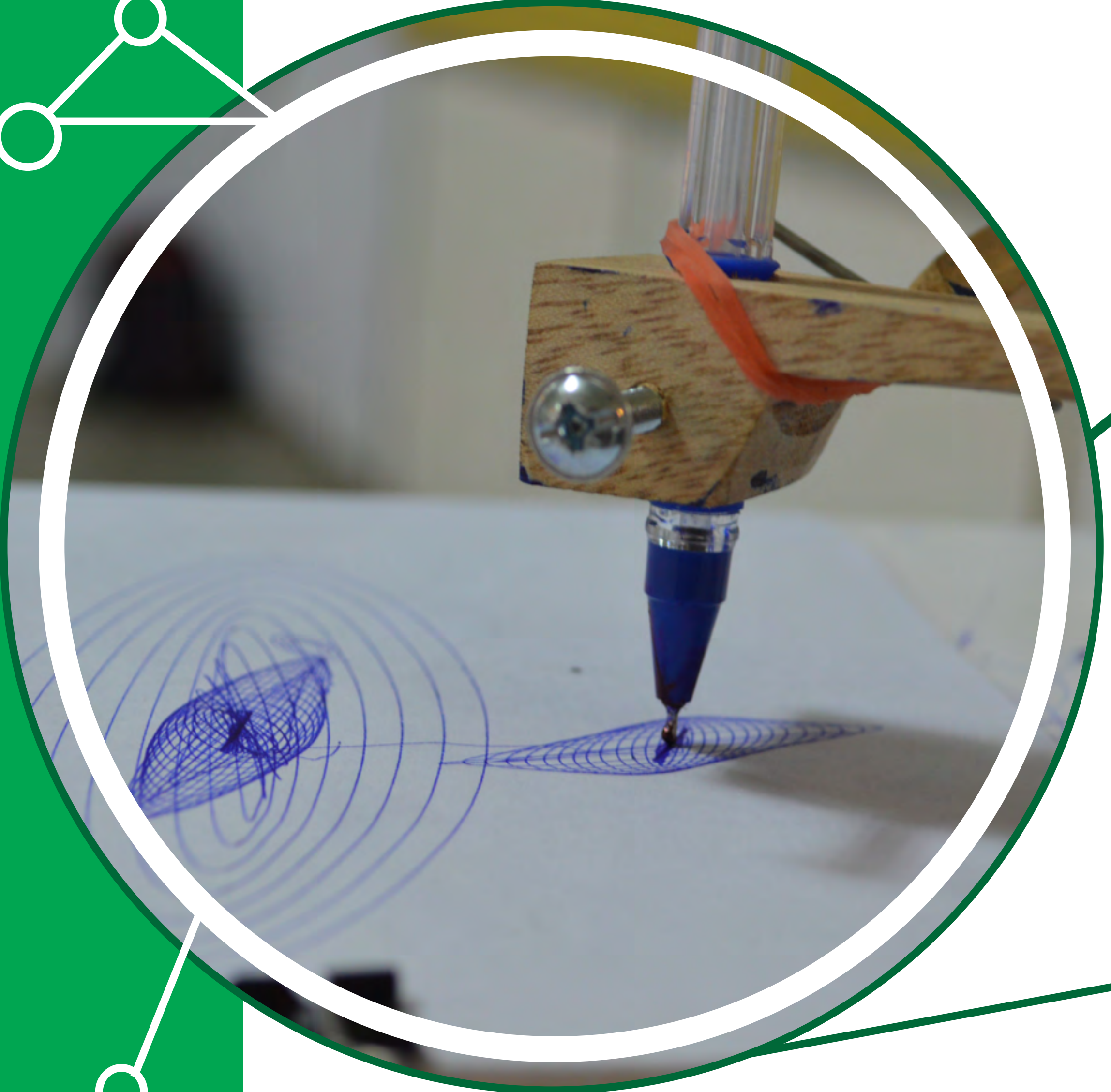

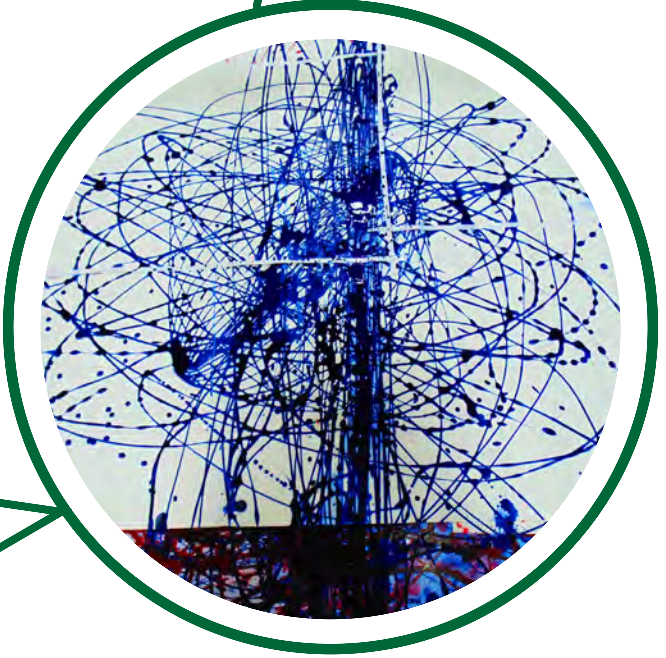

3 oscillations

Regular patterns like those formed when you swing in the park are good examples of oscillations. The main characteristic of oscillations is that one pattern repeats in time, with identical events separated by equally spaced time periods. A canonical example is of the oscillations of a pendulum in the good old wall clocks. These oscillations are exactly like those created by the harmonograph, one of the exhibits described below. But there are other, more complicated oscillations as well - for example, when two pendulums are attached to one another they may create patterns that are not repetitive in time. Such *chaotic* oscillations are illustrated in another exhibit described later.

Where do such system occur? The pendulum in a clock, or the motion of a cricket ball, or of a car engine, or even the earth going around the sun are examples of regular, oscillatory motion.

All these systems are "non-chaotic" in the sense that small changes in the input cause only small changes in the output; If two harmonographs are started with almost the same initial position, they will create almost identical patterns. Thus, even though the pattern may look complicated, it is relatively easy to create the same pattern again and again. We will later see examples of systems which do exactly the opposite: they are called chaotic.

Where is the mathematics? Oscillations are described by their amplitude – how far away from the "centre" or the "sta-

tionary" point does the oscillation go, by a frequency – how many times in a second does it go back and forth, and by a phase – where the oscillation begins. The simplest of oscillations is given by either of the following two equivalent equations:

(1) $\omega(t) = A\sin(2\pi ft + p)$ or $\frac{d^2\omega(t)}{dt^2} + (2\pi f)^2\omega(t) = 0.$

These equations are equivalent in the sense that the solution of the ordinary differential equation on the right is the equation on the left. Here A is the amplitude, f is the frequency, and p is the phase. We will later see a deceptively similar looking equation for a wave.

The power and beauty of mathematics can be illustrated by the following fact: any oscillation can be described by simply a sum of many different oscillations with different values of the amplitude A and the frequencies f. This is a very powerful and useful result that is at the basis of a lot of mathematical ideas as well as their applications, studied under the umbrella of Fourier analysis, named after the French mathematician-physicist Joseph Fourier, who initiated such investigations.

Let us now look at some of the exhibits related to oscillations.