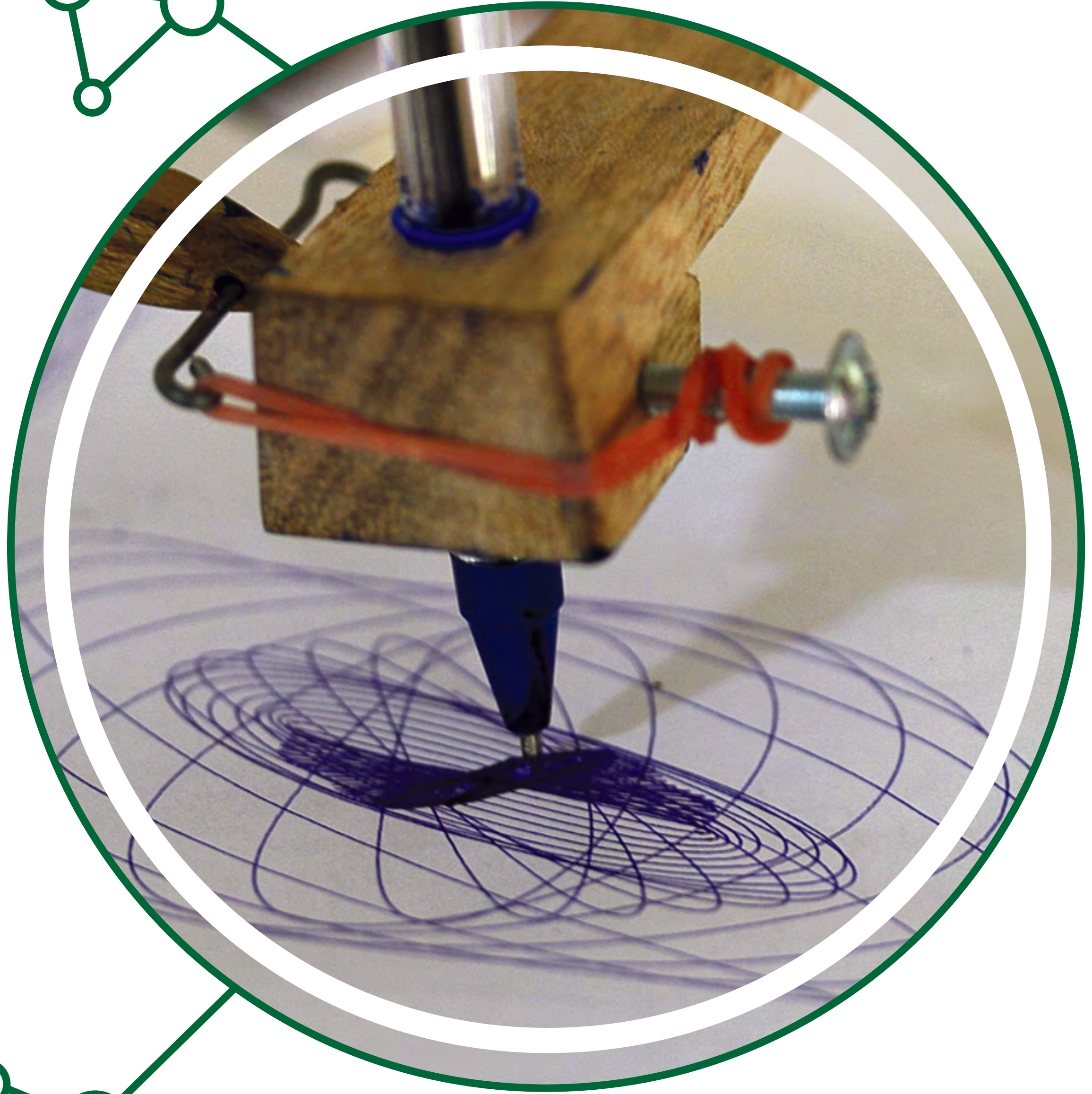
Harmonograph and Lissajous' figures

Harmonograph is essentially a compound pendulum with multiple separate pendulums operating at right angles to each other. The combined oscillations of these pendulums give rise to a pattern called Lissajous' figures. When the different weights hanging from the harmonograph are set swinging at the same time, the observer sees superposition (that is, the sum) of the oscillations of these pendulums. This superposition can give rise to the complex patterns that are drawn on the paper. Different phases and amplitudes of the pendulums give rise to an endless variety of patterns. The visitor starts the pendulums swinging and controls their relative phase.

The exhibit on Lissajous' figures is similar to the harmonograph. The two different oscillations at right angles to each other are achieved by a special arrangement of the "Y" shaped yoke from which the pendulum hangs. When the pendulum is swung, the special pivot enables the same pendulum to operate as two pendulums with different lengths at right angles. As a result, the painting pendulum traces patterns known as Lissajous' figures.

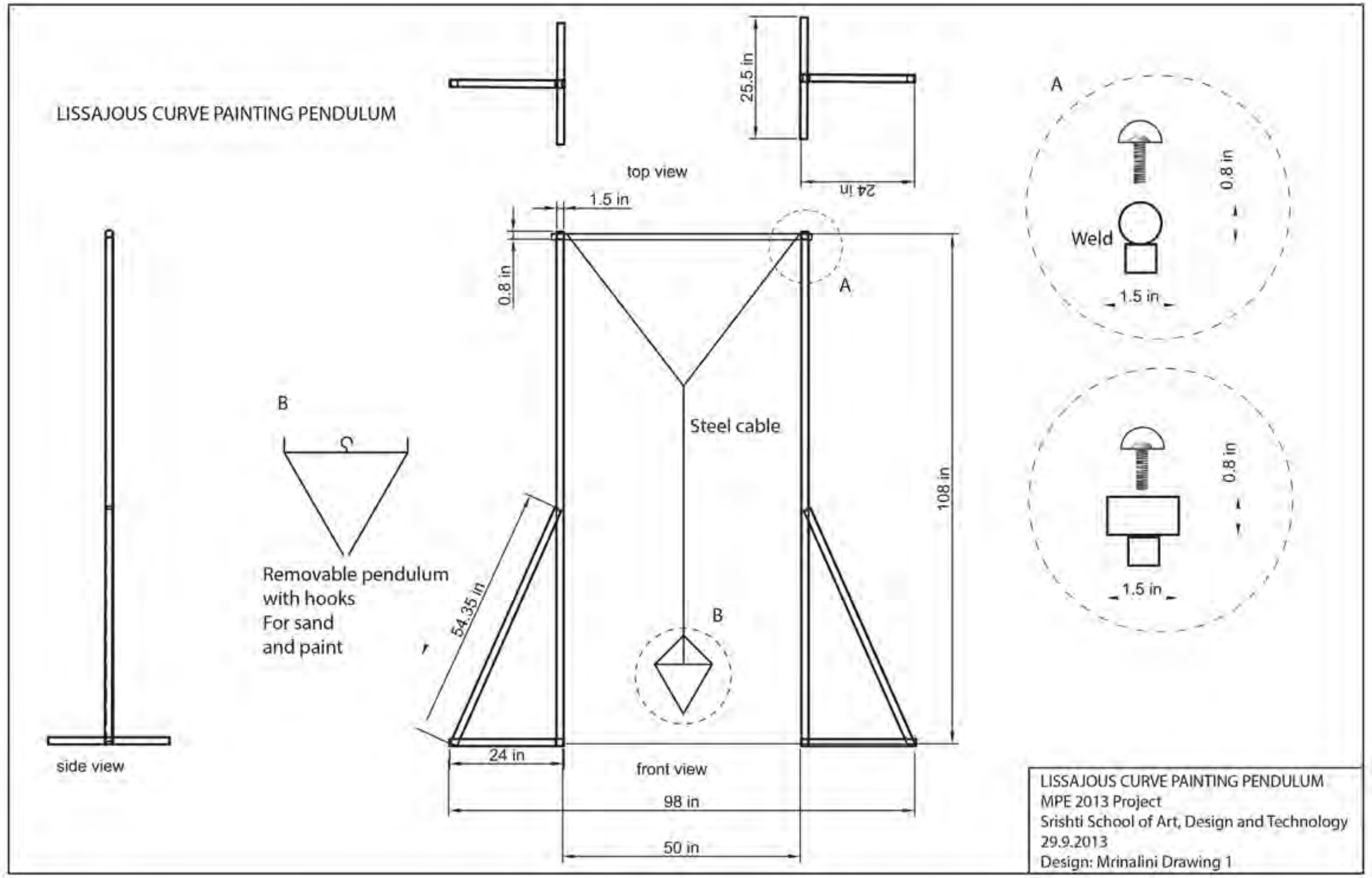


3.1

(2) $x(t) = A \sin(2\pi f_x t + p_x)$ and $y(t) = A_y \sin(2\pi f_y t + p_y)$

Just out of these two simple equations, we can produce all the patterns that you see on the paper below the oscillating pendulum. The equations for the patterns on the harmonograph are similar, but consist of more than one sine term in each of the above equations.[1]

To understand the mathematics behind this, let us call the positions of the pendulum in two perpendicular directions to be $x(t)$ and $y(t)$ - e.g. it could be the distance from the two adjacent sides of the room. Each oscillation has the same form as above:



[1]Some references that discuss this in detail are:
i) http://www.karlsims.com/harmonograph/ ii) http://www.wikihow.com/Make-a-Three-Pendulum-Rotary-Harmonograph