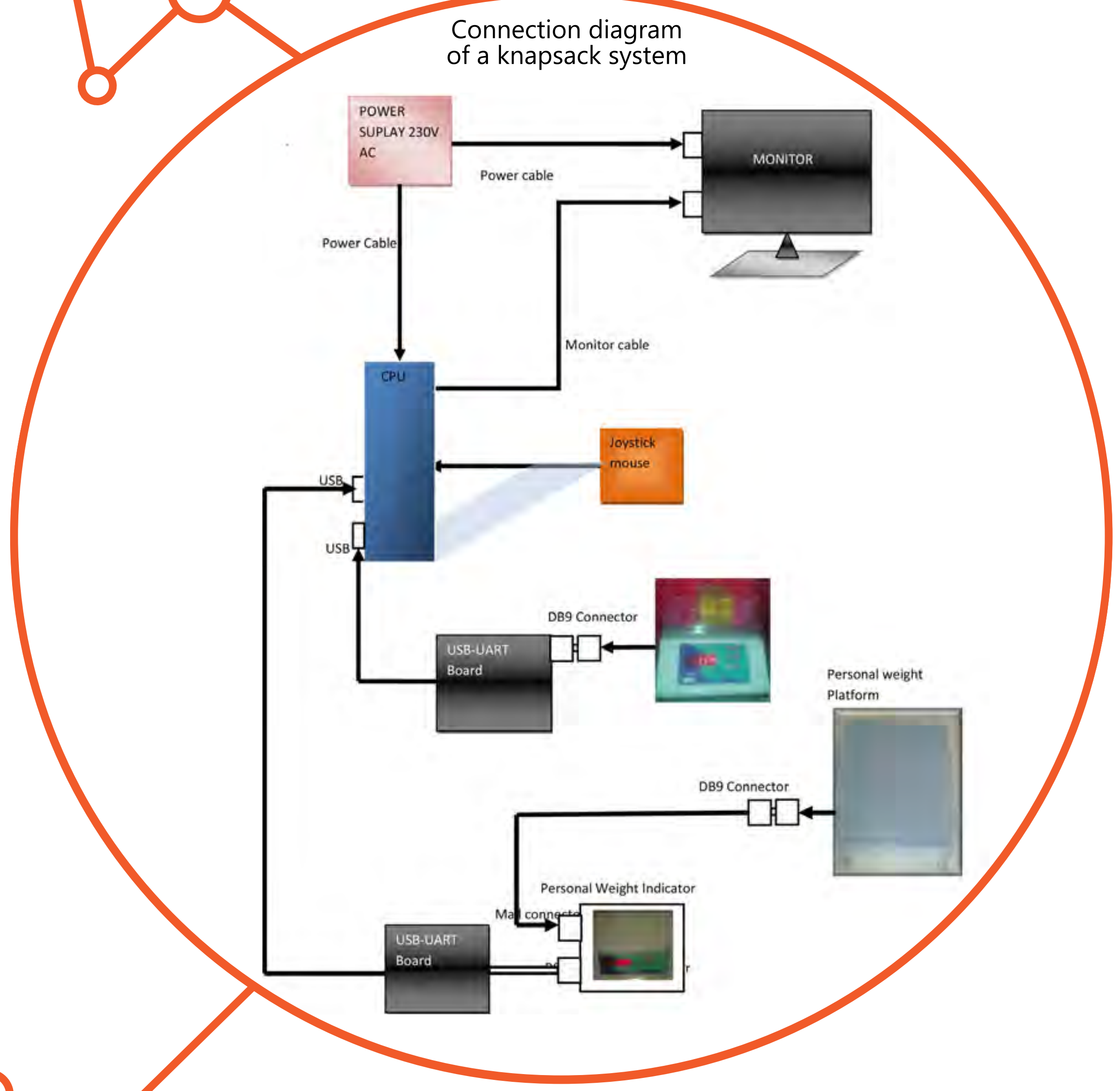
4.1

Knapsack Problem

The original inspiration for the knapsack problem is a puzzle that we are all familiar with. It is one that a vegetable vendor has to solve every day: the vendor goes to the wholesale market to pick up vegetables to sell on a particular day. Of course she cannot carry more than a certain weight, and the total volume cannot exceed the volume of the basket. There are several, at least 10-20, different vegetables she may choose. She knows the profit she can make per kilo of each of the vegetables, and the volume per kilo for each of these as well. Given all this data, she has to choose vegetables that will fit in her basket and are within the weight she can carry, but at the same time will give her maximum profit. This is indeed a complex mathematical problem, but most vendors find a solution for such a problem every day!

More generally, we are all familiar with situations where people have to manage with limited resources. So these problems are of great importance in all walks of life: a homemaker deciding to buy food and groceries for her family for the entire month; the amount of time available to a student to answer a set of questions; scheduling the available manpower to get a job done. In each of these situations, the decision about the best possible manner in which to achieve a stated objective has to be arrived at, while carefully considering the constraints and conditions to be satisfied.

This kind of problem is known as “constrained optimization”: the limits on weight and volume are the “constraints” that cannot be exceeded, whereas “optimization” refers to the aim of trying to maximize the profit. “Knapsack problem” is a colloquial



name for a problem that a thief may face, very similar to that of the vegetable vendor.

A slightly simpler version of this problem is illustrated in this exhibition.

In the exhibit on display, there are several bars each of which has a specific weight and is assigned a value. The visitor stands on the scale and the "constraint" is that he or she cannot choose more than a fifth of his or her body weight. She gets a specified amount of time, say 30 seconds, to choose the weights so as to remain within this weight limit but at the same time maximizing the total value.

Where is the mathematics? Actually there is a whole mathematical field devoted to study these types of problems. The simplest form is as follows. Suppose there are N objects which we will simply number $k = 1, 2, 3, \ldots, N$. Let the k-th object be of weight ω_k with value v_k. Suppose the limit on the maximum weight is W. So we have to choose some of these objects, say p of them $k_1, k_2, \ldots, k_p$, such that

$\omega_{k_1} + \omega_{k_2} + \ldots \omega_{k_p} \leq W$ such that $v_{k_1} + v_{k_2} + \ldots v_{k_p}$ is maximum.

What this really means is that if we had chosen any other set of $\overline{p}$ objects $\overline{k}_1, \overline{k}_2, \ldots k_{\overline{p}}$ with a total weight less than W, then their total value would be less than the set of objects $k_1, k_2, \ldots, k_p$:

$v_{\overline{k}_1} + v_{\overline{k}_2} + \ldots v_{\overline{k}_p} < v_{k_1} + v_{k_2} + \ldots v_{k_p}$.

The set of weights and values that were actually used in this game are recorded in Fig. 4.

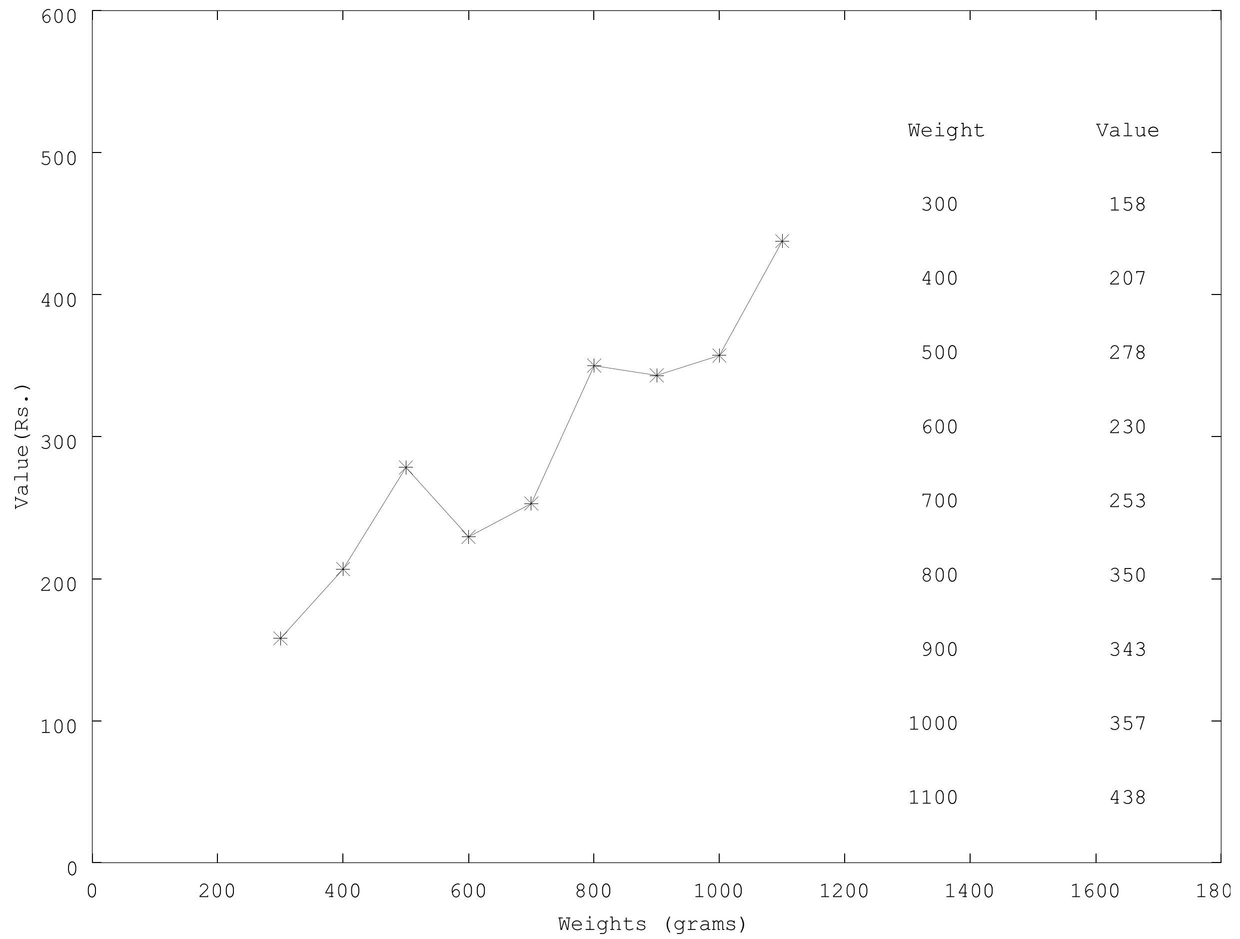


Figure 4. The weights and values assigned to the blocks in the knapsack exhibit