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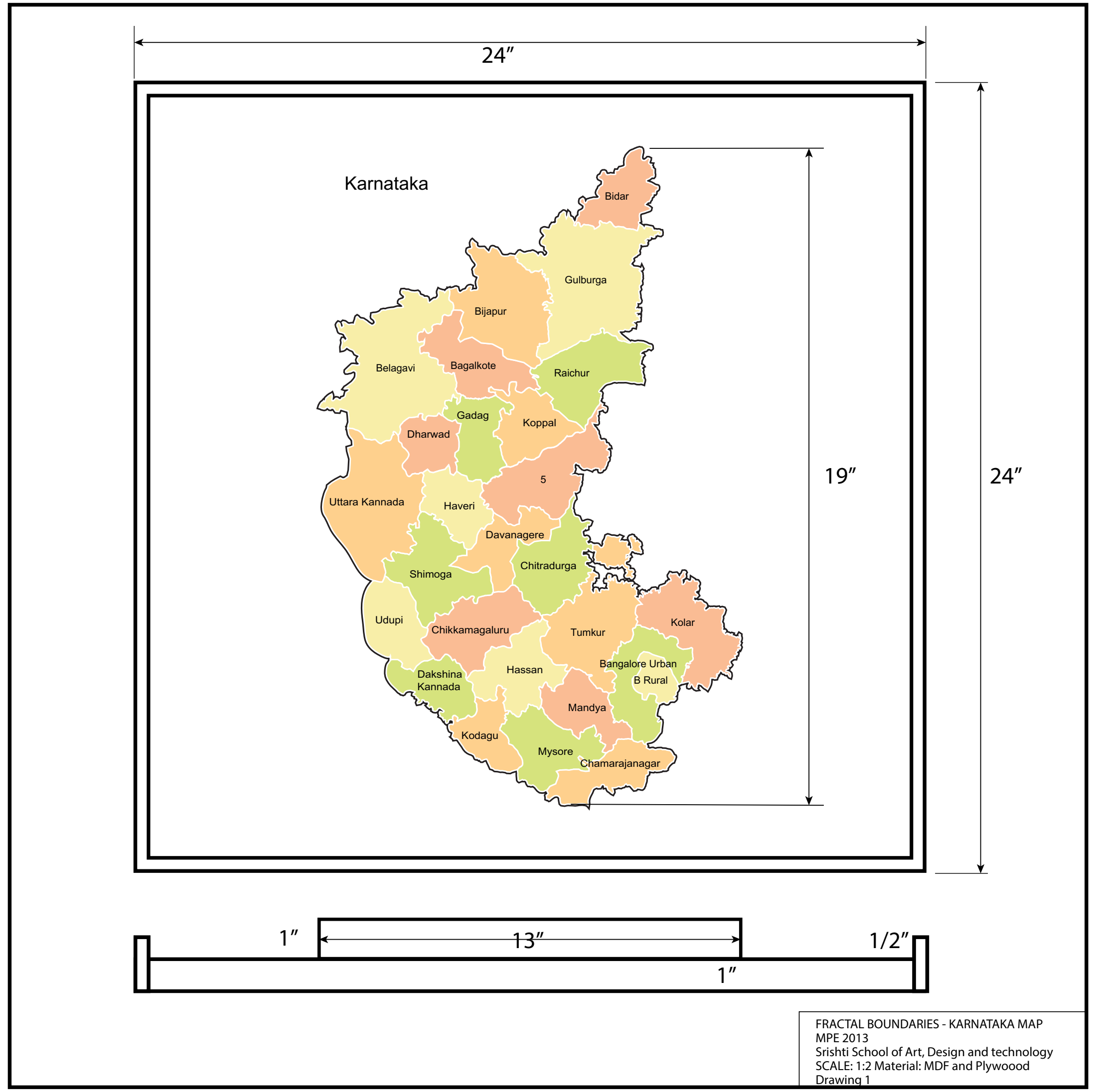
Fractals

This exhibit is designed to illustrate the difference between objects (in this case, curves) which are *fractals* from those which are not. Let us begin with commonly encountered objects which are *not* fractals: a football, the smooth surface of a computer screen or a book, the boundary of a cricket field, train tracks. Indeed, most man made objects are non-fractal objects. On the other hand, approximate fractal properties are observed in many natural objects, such as coastlines, mountain ranges, river networks, lungs, path of lightning, clouds, etc. A quick glance at this list may suggest that fractal properties are related to "non-smoothness" of an object and that is precisely the concept that is captured by the mathematical definition of a fractal, which we will look at after we describe the exhibits.

In the interactive exhibit, we will use rulers with different units. An easy way to make such rulers will be to use beads/thin glass tubes of different diameter/length. Collect few beads/thin glass tubes of diameter/length 1cm, 2cm, 4cm, and 8cm. Use a thread to connect all the beads/thin glass tubes of length 1cm to get a ruler of unit 1cm. Similarly other rulers of unit 2cm, 4cm and 8cm can be made.

There are two experiments to perform.

(1) Experiment 1: This experiment contains a disc and few polygonal shapes with straight edges. Measure the perimeter of each object by wrapping the boundary by different rulers made as above and counting the number of beads/glass tubes. One notices that the perimeter of objects which is obtained by multiplying the number of units by the



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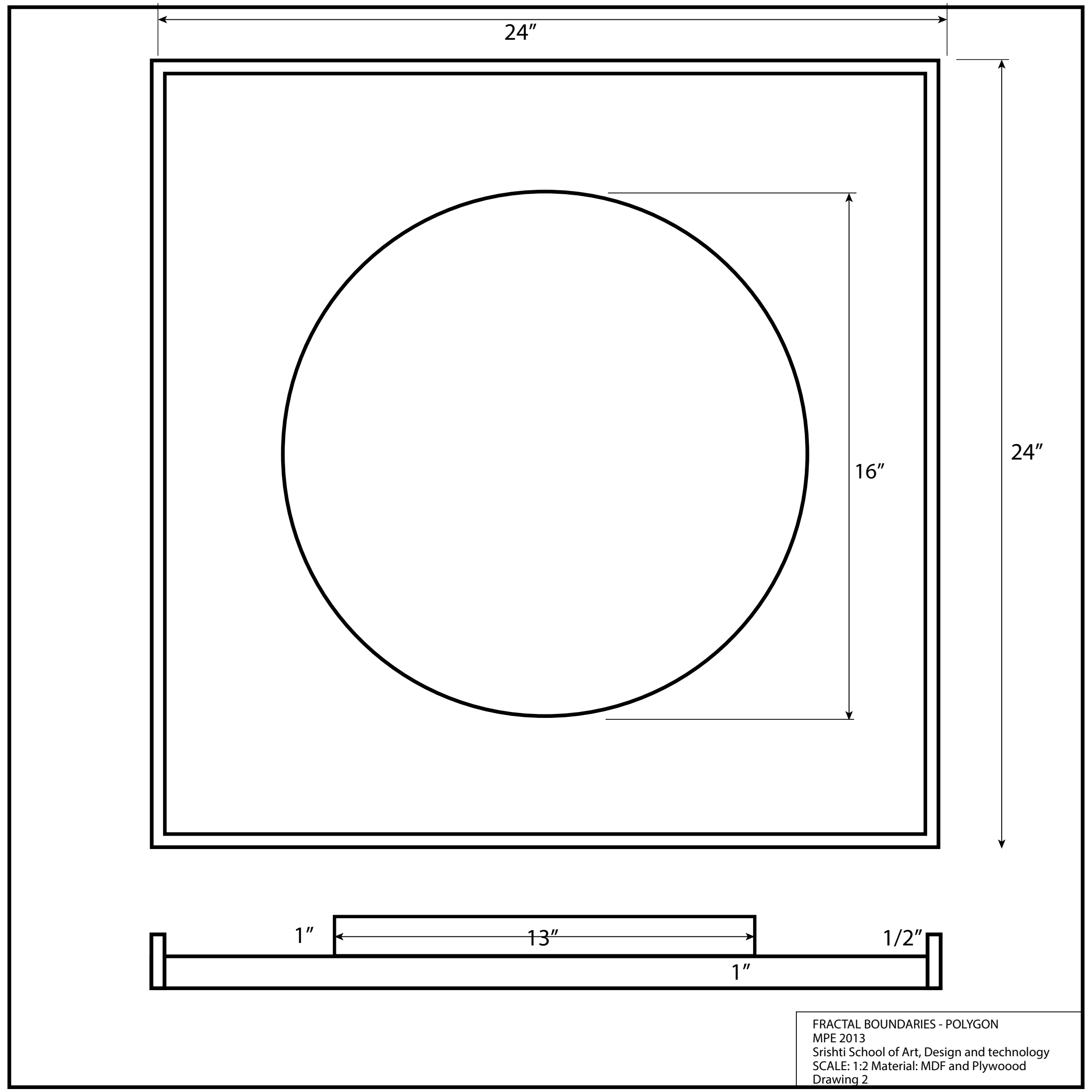
length of one unit remains approximately the same when measured using different rulers.

(2) Experiment 2: This experiment contains objects with highly irregular boundaries. A model of Karnataka (or Norway!) map and snowflake would be ideal choices for this purpose. Repeat the activity of measuring the perimeter with these objects and notice that with different rulers, one gets a different perimeter!

Let's look at the mathematics behind this exhibit to understand the concept of fractals. Let us denote the number of 8cm glass tubes needed to cover the full length of the perimeter of an object by n_8, the number of 4cm tubes needed by n_4, the number of 2cm tubes by n_2 and that of 1cm tubes will be written as n_1. For example, if n_8 is 10, then the perimeter of the object as measured with the 8cm tubes, or mathematically stated as *measured at length scale of 8cm*, would be $8n_8 = 8\times 10 = 80$cm. If $n_2 = 42$, then the perimeter at length scale 2cm would be $42\times 2 = 84$cm.

For non-fractal objects like a square or a triangle or a circle in the experiment 1, you will notice that the number doubles each time we halve the length scale. This relationship is written as: $n_4 = 2n_8$, and $n_2 = 4n_8$ and $n_1 = 8n_8$. So the number of rulers is inversely proportional to the length of the rulers. This is what we will expect intuitively anyway. This can be stated mathematically as follows: the number n_L of tubes with length L needed to cover the full length of the perimeter P would be $n_L = P/L$.



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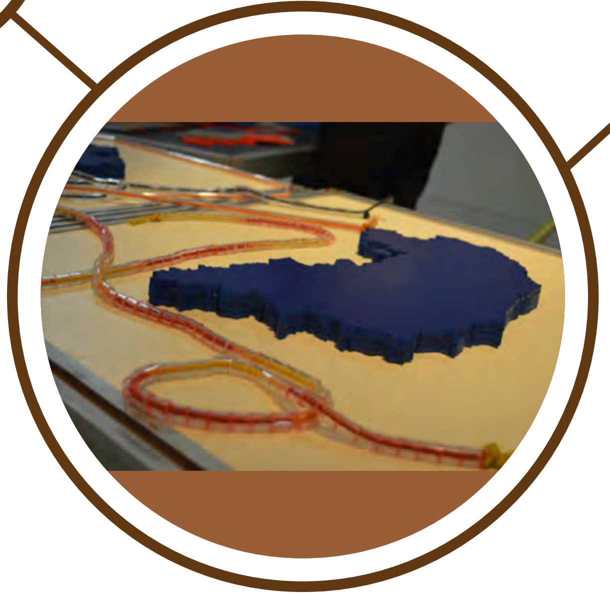

But for a fractal object like a snowflake or Karnataka boundary in experiment 2, the number of rulers needed to measure the perimeter and the length of rulers used do not follow such an intuitive "linear" relationship. In that case, n_4 will be greater than $2n_8$, and similarly, $n_2 > 4n_8$, and $n_1 > 8n_8$, and so on. Indeed, it will be seen that the number of rulers of length L will be given by $n_L = c / L^d$ with d some number between one and two: $1 < d < 2$. Note, that in the case of a non-fractal object, $d = 1$.

In the experiment 1, if the number of rulers n_L is plotted against the scale of the ruler L in a log-log graph, the slope of the line would be -1, but for experiment 2 it will be $-d$. The negative of this slope is called the "dimension" of the perimeter. The perimeter of a snowflake has a dimension which is more than 1 but less than 2, whereas the edge of a circle or a square is of dimension 1.

A similar phenomenon like in experiment 2, which is generally referred as coastline paradox, can be observed when one tries to measure the length of coastline of some countries. These extremely counter-intuitive phenomena of dependence of measurement on the scale is the defining characteristic of the fractal nature of the boundaries of these objects.

Many fractals have another important property called *self similarity* on all scales. If we zoom in, we see the same type of object again: a snowflake seen through different microscopes of different magnifications looks the same! Can you notice whether the snowflake is "self-similar" or not?