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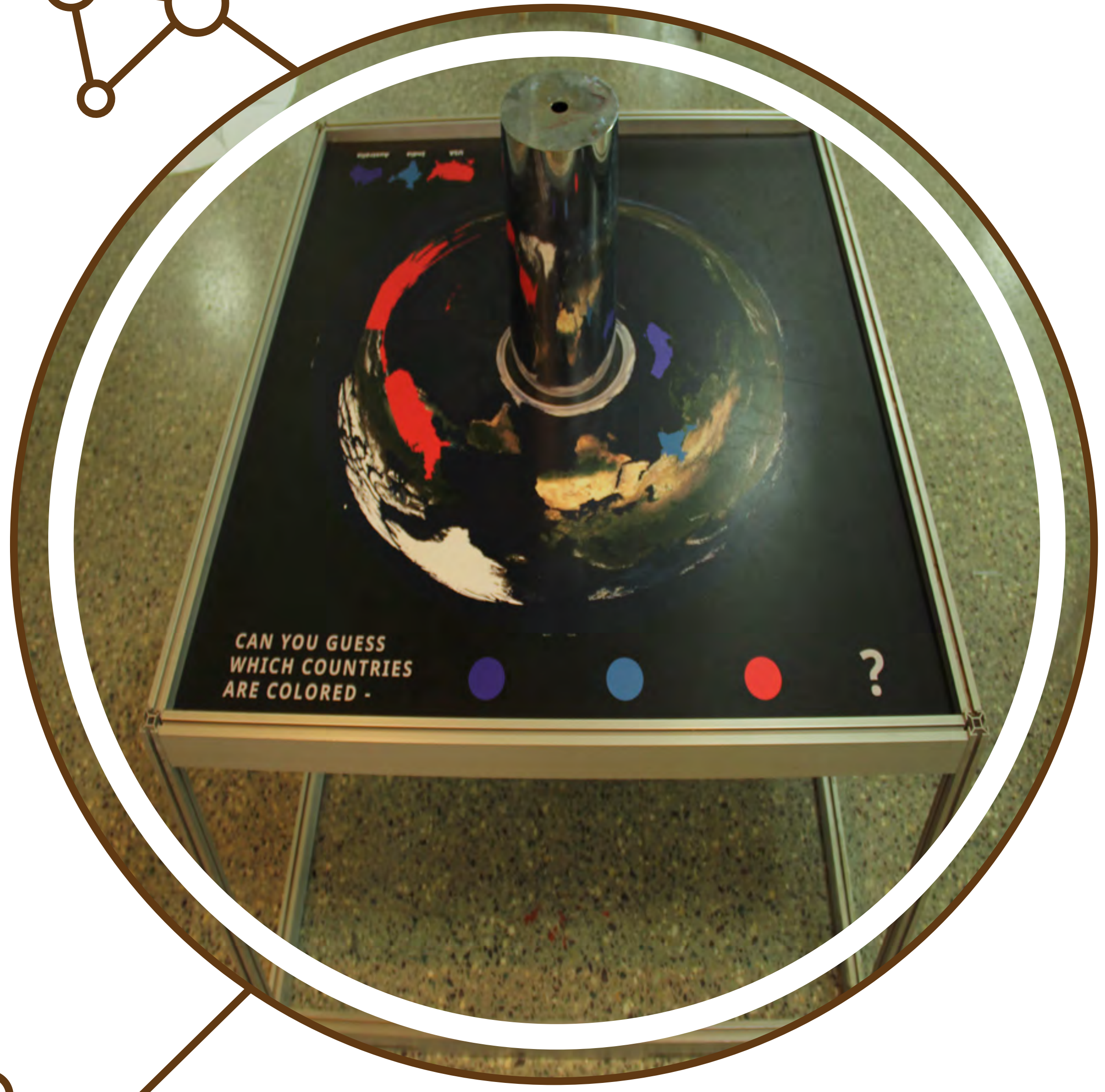
Anamorphic Maps and Foldabale Maps

We can begin with two experiments.

(1) Draw a triangle on a globe. Of course there are readymade triangles on the globe, described by any two longitudes and the equator. Now measure the three angles of this triangle and sum them up. Notice that their sum is never 180 degrees. Why is that so?

(2) Take a large sheet of paper, and try wrapping it around a football or a globe, such that the paper never gets wrinkled. Impossible, right? Next try cutting this sheet of paper into 5-8 pieces (say polygons such as hexagons), and try pasting these polygons on the globe, again without overlapping and no wrinkles. Not possible again! Now cut a large number (100s) of tiny polygons, and then you may notice that it appears possible to achieve this goal. More so because the wrinkles are not going to be visible on such small polygons. How does one understand this?

All the above are manifestations of the same phenomenon: that the earth or the football is not flat, but is spherical and curved! When we want to project such a curved surface onto a flat sheet, it cannot be done without some distortion. E.g. a triangle on the sphere whose angles do not add up to 180^o cannot be projected onto a flat sheet, because if this could be done, its angle would sum up to 180^o, which is clearly a contradiction! The anamorphic map exhibit shows precisely this phenomenon. Notice that the map as seen on the flat surface is highly distorted - the areas, the angles, and the distances are all wrong! But when you see its reflection on the curved surface of a cylinder, it looks just fine. Map-



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makers use the mathematics behind such “reflections” or “projections” in order to make various kinds of maps which reduce one type of distortion or the other – either the angles or the distances or the areas. But no map on a flat piece of paper can reproduce all the three aspects exactly for the whole earth, and that’s a mathematical theorem, so however hard you try, you will never be able to do it!

The foldable maps exhibit illustrates the second phenomenon noted above. If we take large number of small flat polygons, we can approximate the earth surface with much more accuracy than with a small number of large flat polygons. Indeed this property defines the general class of objects called “manifolds” of which the surface of the earth is a familiar example. In general, you can think of “manifolds” as “curved surfaces.” But the power of mathematics is that the same mathematical concepts that describe surfaces (which are two dimensional manifolds) also describe “higher dimensional manifolds” such as the universe in which we live!